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# Functions Measuring the Centrality (or Mediality) of a point in a Network

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## Abstract:

A new theory of functions measuring the centrality (or mediality) of a point in a network is developed.

## Summary:

It is one of the fundamental problems in network theory with applications to study the centrality (or mediality) of a point in a network and properties measuring the centrality (or mediality). In this paper, a new theory of functions measuring the centrality (or mediality) of a point in a network is developed. Kajitani and Maruyama's theory and our old theory are a special case of the theory developed herein.

Consider a connected network  $N$  with vertex set  $V(N)$  and edge set  $E(N)$ . The network may be either directed or undirected. With each edge  $e$  of  $N$ , two kinds of non-negative real numbers  $l(e)$  and  $c(e)$ , called the edge-length and the edge-capacity of  $e$ , respectively, are associated, and with each vertex  $v$  of  $N$ , a non-negative real number  $\sigma(v)$ , called the vertex-weight of  $v$ , is associated. Let  $S$  be a set of points of  $N$  where a point of  $N$  can be either a vertex or an interior point of an edge. For any two points  $s_i$  and  $s_j$  in  $S$ , we define two kinds of non-negative real numbers  $\rho(s_i, s_j)$  and  $\gamma(s_i, s_j)$ , called the di-distance from  $s_i$  to  $s_j$  and the di-capacity from  $s_i$  to  $s_j$ , respectively, by using the underlying graph, edge-lengths and edge-capacities of  $N$ . A typical example of a di-distance from a point  $s_i$  to a point  $s_j$  in  $N$  is the length of a shortest path from  $s_i$  to  $s_j$  in  $N$  and a typical example of

a di-capacity from a point  $s_i$  to a point  $s_j$  in  $N$  is the value of the maximum flow from  $s_i$  to  $s_j$  in  $N$ . The concepts of di-distance and di-capacity are different but are fundamentally important in evaluating the degree of closeness of one point to another. Next, we introduce the concept of monotone modification as a natural unification of network modifications such as adding new edges, coalescing some vertices, shortening edge-lengths and increasing edge-capacities. The monotone modification consists of two kinds of network modifications, called a monotone contraction and a monotone expansion, where the monotone contraction is a network modification with respect to di-distance and the monotone expansion is a network modification with respect to di-capacity. Next, in order to measure the centrality (or mediality) of a point  $r$  in  $N$ , we consider a real-valued function  $f(r, \rho, \gamma, \sigma)$  defined on  $N$  where  $\rho: S \times S \rightarrow \bar{R}_+$ ,  $\gamma: S \times S \rightarrow \bar{R}_+$  and  $\sigma: V(N) \rightarrow \bar{R}_+$  and characterize the centrality of a point  $r$  in  $N$  by using the tendency of the change of the functional value of  $f(r, \rho, \gamma, \sigma)$  under a monotone modification of  $N$ . Next, in the case where we restrict the form of  $f$  to

$$f(r, \rho, \gamma, \sigma) = \sum_{s_i \in V(N) \subseteq S} \psi(\rho(r, s_i), \gamma(r, s_i), \sigma(s_i)),$$

we give a necessary and sufficient condition for  $f$  to be a function measuring the centrality (or mediality) of a point in  $N$  in our sense, and study some relationships between the monotone modification of  $N$  and the convexity or concavity of  $\psi$ .

A precise description of this paper will be included in a full paper of the coming Trans. of IECE of Japan.

#### References:

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